Calculation of electron interaction models in N\textsubscript{2} and O\textsubscript{2}

F. Nicolanti\textsuperscript{a,b,}\textsuperscript{*}, B. Caccia\textsuperscript{c}, A. Cartoni\textsuperscript{d}, D. Emfietzoglou\textsuperscript{e}, R. Faccini\textsuperscript{a,b}, S. Incerti\textsuperscript{f}, I. Kyriakou\textsuperscript{e}, M. Satta\textsuperscript{d,e}, H.N. Tran\textsuperscript{f}, C. Mancini-Terracciano\textsuperscript{a,b}

\textsuperscript{a} Physics Dep., Sapienza U. of Rome, p.le Aldo Moro, 5, 00185, Rome, Italy
\textsuperscript{b} INFN, Sec. of Rome, p.le Aldo Moro, 2, 00185, Rome, Italy
\textsuperscript{c} ISS(IItalian National Institute of Health), V. Regina Elena, 299, 00161, Rome, Italy
\textsuperscript{d} Chemistry Dep., Sapienza U. of Rome, p.le Aldo Moro, 5, 00185, Rome, Italy
\textsuperscript{e} Med. Phys. Lab., Dept of Medicine, University of Ioannina, 45110, Ioannina, Greece
\textsuperscript{f} Université de Bordeaux, CNRS, LP2I Bordeaux, UMR 5797, Chemin du Solarium, 19, 33170, Gradignan, France
\textsuperscript{*} ISMN-CNR, p. Aldo Moro, 7, 00185, Rome, Italy

\textbf{A R T I C L E I N F O}

Keywords:
- Electron cross sections
- Atmosphere ionization
- Particle track structure
- Monte Carlo simulation
- Geant4-DNA

\textbf{A B S T R A C T}

Cosmic rays have the potential to significantly affect the atmospheric composition by increasing the rate and changing the types of chemical reactions through ion production. The amount and states of ionization, and the spatial distribution of ions produced are still open questions for atmospheric models. To precisely estimate these quantities, it is necessary to simulate particle–molecule interactions, down to very low energies. Models enabling such simulations require interaction probabilities over a broad energy range and for all energetically allowed scattering processes.

In this paper, we focus on electron interaction with the two most abundant molecules in the atmosphere, i.e., N\textsubscript{2} and O\textsubscript{2}, as an initial step. A set of elastic and inelastic cross section models for electron transportation in oxygen and nitrogen molecules valid in the energy range 10 eV – 1 MeV, is presented. Comparison is made with available theoretical and experimental data and a reasonable good agreement is observed. Stopping power is calculated and compared with published data to assess the general consistency and reliability of our results.

Good overall agreement is observed, with relative differences lower than 6% with the ESTAR database.

1. Introduction

The emission of polluting molecules and greenhouse gases into the atmosphere represents a global challenge with open scientific questions, mainly related to the approximation grade of chemical–physical processes used in the prevision models of climate change. Although ions play a pivotal role in various atmospheric processes such as ion-induced nucleation, precipitation, and aerosol formation, the influence of cosmic rays and ions on climate is at the dawn of a full understanding and deserves further in-depth investigations [1].

Chemical reaction rates can vary by up to 10 orders of magnitude depending on the ionization state of the involved species [2,3]. Thus, cosmic rays ionization could significantly impact chemical reactions in the atmosphere, considering also that the ions produced are clustered near the primary ray. The amount and state of ionization, as well as the spatial distribution of ions in the atmosphere, are open questions for the atmospheric models, that it is fundamental to investigate also in the very low-energy range [4–6].

Oxygen and nitrogen molecules are the two most prevalent species in Earth’s troposphere and lower stratosphere. Therefore, studying the interaction of cosmic rays with these molecules is crucial for our understanding of various atmospheric phenomena. For instance, electron impact ionization plays a role in the inter-conversion between ozone and oxygen in the atmosphere, while nitrogen is involved in the production of one of the most dangerous greenhouse gases: nitrous oxide [7–9].

Chemical reaction states in collisions with individual molecules represents the elementary process involving the greatest energy transfer and is of central interest in any study concerning the interaction of charged particles with matter. For large energy ranges, it is also the most likely process.

Event-by-event simulations are a powerful tool for studying the details of radiation-induced effect at the molecular level. Nowadays, several existing Monte Carlo track-structure (MC-TS) codes fulfill this purpose. One of these is Geant4-DNA [10–13], an extension of Geant4 (GEometry ANd Tracking) [14–16] which is the most widely used toolkit for performing MC simulations of radiation–matter interactions. Geant4-DNA makes it possible to explicitly simulate every single electromagnetic particle interaction down to low energy (about 10 eV), as
well as diffusion and chemical reactions, on some specific materials of interest in radiobiology [17,18].

The aim of this work is to provide a comprehensive and reliable set of electron impact electromagnetic interaction models for O₂ and N₂ molecules down to the 10 eV scale, for use in such simulation code. This will allow to accurately simulate the interaction of low-energy secondary radiation with molecules, the exact concentration of ions produced, their spatial distribution, and the ionization state.

We selected a model for each relevant electromagnetic electron impact interaction process in the energy range 10 eV – 1 MeV, including ionization, electronic excitation, and elastic scattering. Given the computational constraints of a simulation code, we focused on choosing calculation methods that are both sustainable and efficient. In this regard, models that produce an analytical expression for the cross section and a good compromise between accuracy and computational time were preferred. The physics models presented here can be easily adapted to be used for molecules in the gas phase and further work is planned to extend them to other species of climate interest. One of these is the trace gas SO₂, studied at CERN in the CLOUD experiment [19,20], which has proven to have large effects on ozone chemistry.

The selected models are briefly described in Section 2. In Section 3, we present the benchmark of our models implementation. Specifically, in Section 3.1, we show the partial cross sections obtained for each process and compare them with experimental data or other calculations. In Section 3.2, we provide a final validation by comparing the calculated stopping power, obtained using the selected models, with semi-empirical results and ab-initio calculations. The stopping power is also compared with the ICRU (International Commission on Radiation Units and Measurements) recommended values calculated by ESTAR available down to 1 keV, which are a good benchmark for MC calculations. The ESTAR values have uncertainties ranging from 1% to 2% (in low-Z materials), for energies higher than 10 keV. These uncertainties grow up to 10% at 1 keV, due to the omission of shell corrections [21].

2. Description of physics models

2.1. Ionization

Electron impact ionization is based on the Relativistic Binary Encounter Bethe (RBEB) model [22], which combines the relativistic Mott cross section, known as the Möller cross section, with the relativistic version of the Bethe cross section. It represents the high-energy extension of the corresponding Binary Encounter Bethe model developed by Kim and Rudd [23].

Within the framework of this model, the energy differential cross section (DCS) \( \frac{d\sigma^{\text{MO}}}{dw} \) with the energy of the ejected electron \( W \) at a given incident energy \( T \) for a molecular orbital (MO), can be written as

\[
\frac{d\sigma^{\text{MO}}}{dw} = \frac{4\pi a_0^2 a_s^2 N}{(\beta_i^2 + \beta_o^2 + \beta_e^2) 2\beta_e} \times \left( -\frac{1}{t+1} \left( \frac{1}{w+1} + \frac{1}{1-w} \right) - \frac{1}{t-w} \right) \times \left( \frac{1}{t+1} + \frac{1}{t-w} + \frac{\beta_e}{(1+t'2)^2} \right) \times \left( 1 + \frac{1}{(w+1)3} + \frac{1}{(t-w)3} \right)
\]

(1)

where

\[ t = \frac{T}{B}, \quad w = \frac{W}{B}, \quad \beta_e = \frac{E_e}{B}, \quad \beta_i = \frac{E_i}{B}, \quad \beta_o = \frac{E_o}{B} \]

The RBEB formula depends only on the three input parameters, i.e. the binding energy (\( B \)), the mean kinetic energy (\( U \)) and the occupancy number (\( N \)) of each molecular orbital. The simple analytical form in Eq. (1) is ideally suited for modeling applications and Monte Carlo simulation as it allows energy loss to be randomly sampled during an ionization event without the need for lengthy cross sections tables [24].

This model is valid for electron energies that are significantly higher than the binding energy of the target electron, as it is based on the First Born Approximation. Nevertheless, for many stable molecules including N₂ and O₂, it yields ionization cross sections that are in good agreement in both magnitude (with deviations of 15% or less at the peak) and shape from each shell ionization threshold onwards [25,26]. We have imposed the high-energy limit to 1 MeV, as for higher energies other relativistic effects, such as the density effect, must be considered.

In the RBEB model the scattering angle of the primary electron and the ejected angle of the secondary electron are assumed to be isotropic. This approximation can be reduced by introducing a sampling of both angles determined by the kinematics of binary collisions [27].

The ionization cross section for each molecular orbital is given by integration of Eq. (1) up to the maximum energy of the ejected electron \( W_{\text{max}} = (T - B)/2 \), namely:

\[
s_i^{\text{MO}} = \frac{4\pi a_0^2 a_s^2 N}{(\beta_i^2 + \beta_o^2 + \beta_e^2)2\beta_e} \times \left\{ \frac{1}{2} \ln \left( \frac{\beta_e^2}{1-\beta_i^2} \right) - \beta_e^2 - \ln(2\beta_e') \right\} \times \left( 1 - \frac{1}{t^2} \right) + \frac{1}{t + 1 + 2t'} \ln \left( \frac{1 + 2t'}{t + 1 + (1+t'/2)} + \frac{\beta_e^2}{(1+t'/2)^2} \times \frac{t' - 1}{2} \right) \right\}
\]

(2)

For single ionization of inner k-shells which are subject to stronger nuclear attraction, we use the averaged RBEB formula [22,28]:

\[
s_i^{\text{k-shell}} = \frac{1}{2} \left( 1 + \frac{\beta_e^2 + \beta_o^2 + \beta_i^2}{\beta_i^2} \right) \times s_i^{\text{MO}}
\]

(3)

For oxygen molecules there are five outer shells and the inner K-shell of the oxygen atom, while for nitrogen molecules there are four outer shells plus the inner K-shell of the nitrogen atom. For each outer shell, the binding energies and the mean kinetic energies are from Hwang et al. [29], while K-shells parameters for diatomic molecules are from Jolly et al. [30] (Table 1).

2.2. Elastic scattering

Elastic scattering, although involving only minimal energy loss, strongly influences the accuracy of the spatial distribution of energy deposition.

To calculate the differential and integral elastic cross sections we used the IAM-SCAR method, which is based on the Independent Atom Model representation (IAM) [31] complemented with a Screening-Corrected Additivity Rule (SCAR) [32,33]. This method has already been extensively employed to calculate electron-scattering cross sections for a wide variety of molecular targets, over a broad energy range [34–38].

Under the IAM approximation, the scattering from a molecule is described by the direct and spin-flip scattering amplitudes:

\[
F(\theta) \approx \sum f_i(\theta)e^{\text{i}q \cdot r} \quad \text{and} \quad G(\theta) \approx \sum g_i(\theta)e^{\text{i}q \cdot r}
\]

(4)

where \( q = k_f - k_i \) is the momentum transfer, \( r \) are the atomic positions, and \( f_i(\theta) \) and \( g_i(\theta) \) are the atomic scattering amplitudes. By averaging the modulus squared of the scattering amplitudes \( |F(\theta)|^2 \) and \( |G(\theta)|^2 \)
over all molecule orientations [2, 6], we obtain the differential elastic cross section:

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{ij} \left[ |f_i(\theta)f_j^*(\theta) + g_i(\theta)g_j^*(\theta)|^2 - |f_i(\theta)|^2|g_j(\theta)|^2 \right] \sin(\theta) d\Omega
\]

where \( q \equiv |q| = 2k \sin(\theta/2) \) and \( r_{ij} \) is the distance between the \( i \) and \( j \) atoms. By integrating (5), the total molecular cross section can be written as:

\[
\sigma_{\text{FAM}} = \sum_i \sigma_i + \sigma_{\text{interference}} = \sigma^{AR} + \sigma_{\text{interference}}
\]

The first term \( \sigma^{AR} \) corresponds to the direct sum of atomic cross sections and it is equivalent to the molecular cross sections according to the Additivity Rule (AR) [39]. The second term \( \sigma_{\text{interference}} \) represents the interference contribution between the two single scattering events. Its main effect is to increase the differential cross sections at small scattering angles (\( \theta < 30^\circ \)), leading to an overall increase in the integral cross section values. It must be noted that the second contribution would not be present in (6) if this expression were directly obtained from (4) by applying the optical theorem. Thus, differential cross sections should be renormalized to avoid inherent contradictions [35,36].

The above expressions are applicable for independent scattering from each atom and they are only valid for large interatomic distances compared to the wavelength associated with the incident electron (\(< 200 \text{ eV for N}_2 \) and \( \text{O}_2 \)). To extend their applicability to lower energies, the SCAR method introduces screening coefficients (\( s_\lambda \)) in Eqs. (5)–(6). These coefficients have the effect to reduce the contribution of each atom to the overall molecular cross section (\( 0 \leq s_\lambda \leq 1 \)) in the low energy range. Further details on these coefficients can be found elsewhere [32,33,40].

Atomic scattering amplitudes and cross sections in (5) were calculated using the ELSEPA (Elastic Scattering of Atoms and Positrons by Neutral Atoms) code developed by Salvat et al. [41], which uses the Dirac partial-wave approach including relativistic corrections to calculate the electron elastic scattering by a local central interaction potential representing atoms. To perform the calculation in the optical potential formalism, we considered the Fermi nucleus distribution, the Dirac–Fock electron distribution, the Furness–McCarthy exchange potential, the correlation-polarization potential with the local density approximation, and the LDA absorption potential, described in detail in Ref. [41].

In our study, we observed that including the interference terms gives good agreement with reference data at small scattering angles (< 30°), but results in a significant overestimation of integral cross sections (as shown later in Fig. 5). Consequently, we decided to employ the simple incoherent sum of atomic scattering amplitudes, following the AR approximation. This approach not only eliminates the need for renormalization to satisfy the optical theorem, but also significantly enhances the agreement between the total cross sections (TCSs) and the experimental data, as previously stated in [42]. Nevertheless, to enhance the accuracy of our calculations, we introduced slight adjustments to the free parameters of the scattering potential, specifically targeting an increase in the differential cross sections at small scattering angles. The two parameters involved are \( b_{\text{pol}} \) and \( A_{\text{abs}} \), included in the correlation polarization potential and the absorption potential, respectively. The default values used in ELSEPA have been validated for noble gases and mercury, but they can be modified to better match experimental data. Notably, above the ionization threshold, a higher absorption strength increases the DCSs at small scattering angles while decreasing them at intermediate and large angles. As for the \( b_{\text{pol}} \) parameter, the DCSs at small angles are the highest when \( b_{\text{pol}} \) is the lowest and decrease with increasing \( b_{\text{pol}} \).

By examining the impact of these two parameters on the DCS, we qualitative determined optimized values for \( b_{\text{pol}} \) and \( A_{\text{abs}} \). Specifically, for energies below 500 eV, the value of \( b_{\text{pol}} \) is fixed at 0.01. As the energy approaches 1 keV, it linearly increases from 0.01 to 6.6. Beyond 1 keV, the expression \( \sqrt{E/\text{eV}} \text{eV}^{\text{2kV}} \) governs the behavior of \( b_{\text{pol}} \) for both molecules. Regarding the \( A_{\text{abs}} \) parameters for \( \text{N}_2 \), it decreased linearly from 2.5 to 12 eV to 2 at 400 eV. For \( \text{O}_2 \), \( A_{\text{abs}} \) transitions from 2 at 12 eV to 3 at 70 eV, and then decreases back to 2 at 300 eV. For higher energies, \( A_{\text{abs}} \) is set to 2 for both molecules, which corresponds to the default value proposed by ELSEPA.

It should be made clear that the cross sections for each constituent atom are calculated using the known first optically allowed excitation threshold [43] and atomic polarizability [44] of that atom, prior to the application of the screening corrected additivity rule. The bond distances are taken from the pubchem database of 3D molecular structures [37]. For \( \text{N}_2 \), the bond length is 1.12 Å, while for \( \text{O}_2 \), it is 1.23 Å [45].

### 2.3. Electronic excitation

The third process which is of great importance during electron slowing down in the atmosphere is the excitation of molecules by electron impact. This process plays an important role in determining the internal energy and state distribution of the gaseous molecules in the atmosphere, and is mainly responsible for the increase of the mean energy loss in the low energy region.

The treatment of excitation process in oxygen and nitrogen is based on the relativistic formulae of Porter et al. [46]. For optically forbidden discrete excitations the cross section \( \sigma^{\text{exc}}_j \) to a state \( j \) of electrons at energy \( T \) is given by:

\[
\sigma^{\text{exc,forbidden}}_j = \frac{q_j A_j (2W_j/m\mu^2)}{(m\mu^2/2W_j)^{2/3}}
\]
where
\[
\phi (2W_j/m\beta^2v^2) = \left[1 - \left(\frac{2W_j}{m\beta^2v^2}\right)\right]^\frac{\beta}{2} - 1
\]
is a distortion factor and allows for variations from the asymptotic Bethe formula at low energies, \(q_0 = 4\alpha_0^2R^2\) and has the numerical value of 6.513 \times 10^{-14} eV^2 cm^2, \(\alpha_0\) being the Bohr radius and \(R\) the Rydberg energy. \(W_j\) is the threshold excitation energy of the \(j\)th states, and \(A, \alpha, \beta, \gamma\) are four adjustable parameters.

The second form used to represent the cross section of discrete allowed excitations and for the excitation of Rydberg states is more consistent with the asymptotic form of Born–Bethe theory, and is calculated according to:

\[
\sigma_j^{\text{exc,allowed}} = \frac{q_0A\phi (2W_j/m\beta^2v^2)}{(m\beta^2v^2/2W_j)^2} \ln \left[4 \left(\frac{m\beta^2v^2}{2W_j}\right) C_j \left(1 - \frac{1}{\beta^2}\right) + c - \beta^2 \right]
\]

where \(\phi (2W_j/m\beta^2v^2)\) is given by
\[
\phi (2W_j/m\beta^2v^2) = \left[1 - \left(\frac{2W_j}{m\beta^2v^2}\right)\right]^\frac{\beta}{2} - 1
\]
with
\[
\Theta \left(\frac{m\beta^2v^2}{2W_j} - 1\right) = \begin{cases} 1, & m\beta^2v^2/2 \geq W_j \\ 0, & m\beta^2v^2/2 < W_j \end{cases}
\]

We determined the free parameters in Eqs. (7)-(8) \((A, \alpha, \beta, \gamma)\) for most of the excitation states through a non-linear least squares analysis of various experimental data sets, following the procedure outlined in [47]. In cases where literature data were unavailable, the values for the excitation parameters were taken from the study by Porter et al. [46]. The cross section parameters and formulas for the excitation of Rydberg states were also taken from the publication by Porter et al. Their formulas employ a method to calculate the \(A\) and \(W_j\) parameters for each Rydberg series, that takes into account the principal quantum number and the quantum defect associated with the specific state.

The excitation of three possible allowed states \((B^3\Sigma^+_v, B^3\Pi^+_v, A^3\Sigma^+_v)\), three forbidden states \((b^1\Sigma^+_v, a^3\Pi^+_v, A^3\Pi^+_v)\), plus 21 Rydberg states (7 with principal quantum number equal to 3, 7 cumulative for all higher lying members), was taken into account in the present study for oxygen.

For molecular nitrogen three allowed levels \((b^1\Pi^+_v, b^3\Sigma^+_v, a^1\Pi^+_v)\), twelve forbidden levels \((vib \ 1–3, \ vib \ 4–8, A^3\Pi^+_v, B^3\Pi^+_v, B^3\Sigma^+_v, d^1\Sigma^+_v, \ w^3\Delta^+_v, C^3\Pi^+_v, E^3\Sigma^+_v, a^1\Pi^+_v, \ d^3\Pi^+_v)\), plus 18 Rydberg states (6 with principal quantum number equal to 3, 6 with principal quantum number equal to 4, and 6 cumulative for all higher lying members), have been included. The values of the forbidden and allowed excitation parameters used in the present study and partially based on Porter et al. are listed in Table 2. For the parameters related to the excitation of Rydberg states, please refer to [46].

For the use of such a model for simulation purposes, the excitation of Rydberg-like states that often leads to auto-ionization needs to be considered. Following the recommendations of Stolarski [50] and Watson [51], a 50% probability of auto-ionization should be assumed when the excitation energy of a Rydberg state is greater than the ionization threshold for the material.

### 3. Results and discussion

We implemented the described models in C++, with the aim of interfacing them with Geant4-DNA in the near future. The computation of ionization and excitation cross sections is performed using their analytical formulations. As for elastic cross sections, we have employed interpolated cross section data tables, following the approach commonly used in particle transport codes.

#### 3.1. Cross sections

#### 3.1.1. Differential ionization cross section

For a given incident energy, the sum of Eq. (2) over all molecular orbitals gives the energy differential ionization cross-section. This has been evaluated at impinging electron energies varying from 10 eV to 1 MeV employing the RBBE formulation. In Fig. 1, the results are compared with the experimental cross sections from Opal et al. [52], Shyn [53,54], DuBois and Rudd [55], and the theoretical data from Pal et al. [56], computed by using the Jain–Khare semiempirical approach [57]. The differential cross sections reveal a good agreement with the experimental values and better than those determined from the Jain–Khare method. The main differences are observed for an incident
BEf-scaling results [75–77] for the dipole-allowed transitions. (MagBoltz, versions 8.9 and later) [73,74] and the semi-empirical obtained through data transcribed from S.F. Biagi’s FORTRAN code in the intermediate-high energy range, the asymptotic dependence was recommended by Itikawa [48,49]. In the absence of experimental data at low energies and for the lower states is mainly based on the values empirical data used for the fitting procedure. The cross sections trend in the present study are shown, along with experimental or semi-

3.1.3. Excitation cross sections

In Fig. 4 the excitation cross sections for each of the levels included in the present study are shown, along with experimental or semi-empirical data used for the fitting procedure. The cross sections trend at low energies and for the lower states is mainly based on the values recommended by Itikawa [48,49]. In the absence of experimental data in the intermediate-high energy range, the asymptotic dependence was obtained through data transcribed from S.F. Biagi’s FORTRAN code (MagBoltz, versions 8.9 and later) [73,74] and the semi-empirical BEf-scaling results [75–77] for the dipole-allowed transitions.

3.1.4. Total cross sections

The total scattering cross section \( \sigma_{\text{tot}} \) of electrons at energy \( T \) in nitrogen and oxygen was calculated as \( \sigma_{\text{tot}} = \sigma_{\text{ion}} + \sigma_{\text{el}} + \sigma_{\text{exc}} \) and can be compared in Fig. 5 with the recommended values by Itikawa and other cross section results. The individual contributions of ionization, excitation, and elastic processes are presented in solid lines, alongside experimental data and semi-empirical calculations (further details in the plot legend).

As expected, the excitation \( \sigma_{\text{exc}} \) and ionization \( \sigma_{\text{ion}} \) cross sections exhibit similar energy dependence for energies above approximately 200 eV, but \( \sigma_{\text{exc}} \) increases with decreasing energy due to the contribution of excitation to optically forbidden states. A glance at the figures shows that the high values of the elastic cross section, compared to those of other interaction effects, are remarkable at energies smaller than 100 eV.

The elastic cross section results demonstrate good agreement with all experimental data sets for \( \text{N}_2 \). For oxygen, small deviations are noticed for energies below 60 eV [48,81]. Nevertheless, these deviations are still within the range of Itakawa’s experimental uncertainties (±20%, not displayed in the plot).

The results obtained from the IAM-SCAR model are included for comparison in Fig. 5. As previously mentioned, the inclusion of the interference term in (6) leads to an overall increase in the integral elastic cross section across the entire energy range. Specifically, this enhancement is about 26% and 24% at 100 eV for \( \text{N}_2 \) and \( \text{O}_2 \), respectively, and decreases to 15% and 10% at 10 keV. It is worth noting that the experimental values from Iga et al. [58], and Daimon et al. [59] for \( \text{O}_2 \) demonstrate better agreement with the IAM-SCAR model compared to the AR-SCAR approximation. This overestimation of the TCS when compared to more recent experimental data is also reflected in their small-angle DCs in Fig. 3, that show higher values in comparison to the predictions of the AR-SCAR model.

In Fig. 5 the excitation cross sections obtained in the present study are compared with those obtained using Porter’s default parameters. For oxygen, the fitting procedure based on the most recent cross section
measurements allows for a better description of the cross section at very low energies, while maintaining the same behavior at intermediate and high energies. As for nitrogen, our results show lower values compared to Porter’s results, but they are in better agreement with the comprehensive set of surveyed excitation cross sections by Majeed and Strickland [82].

The total and partial cross sections for nitrogen obtained in the present study are compared with Grosswendt’s results (dash–dot lines in Fig. 5), whose models are extensively described in [83–85]. TCSs are in good agreement with each other for energies higher than 1 keV, although the individual contributions of different processes are different. Grosswendt employs the non-relativistic version of the RBEB, which leads to an underestimation of the ionization cross section at high energies, and an empirical screened Rutherford formula to describe the elastic process. The excitation cross section used by Grosswendt is based on the formulas and cross section parameters of Porter et al. They introduced modifications to Porter’s parameter to enhance excitation cross sections across the entire energy range, aiming to improve agreement with experimental TCS measurements. In line with this goal, they incorporated an extra excitation contribution to the cross section shape, which results in the 20 eV peak [83]. It is worth noting that the Grosswendt’s cross-sections for N₂, already used in the PTra code developed at PTB, have recently been implemented in the Geant4-DNA toolkit [86].

The comparison of our results with other TCS experimental data and theoretical calculations demonstrates an overall good agreement. However, it is important to highlight some additional considerations. Firstly, the RBEB model employed in this study tends to overestimate the ionization cross section, particularly at lower energies and near the peak. This overestimation may arise from the approximation used, which does not account for differential oscillator strengths, as previously discussed by Bug et al. [85]. Secondly, the contribution of the excitation cross section also introduces a degree of uncertainty. For nitrogen, Itikawa does not provide recommended cross sections for the excitation of higher allowed states (i.e., those with thresholds above...
12.5 eV), which exhibit large cross sections even at high electron energies. Due to the limited availability of measured values, the Rydberg cross section is also a major source of uncertainty. Lastly, it is important to note that most of the recommended data for forbidden excitation processes have large uncertainties, typically ranging from 25% to 40%. This reflects significant differences in the differential cross sections measured by different research groups.

### 3.2. Stopping power

To test the consistency of the presented interaction cross sections, we evaluated the electron stopping power from the analytical cross sections' models.

Assuming that an electron of initial energy $E$ loses its energy only through exciting and ionizing collisions, the analytical mass stopping power can be written as

$$-(dE/dx)_{\text{ion}} = -n/\rho \sum_i \int_{B_i} \frac{E+B_i}{2} \frac{d\sigma_{\text{ion}}}{dW}(E)dW$$

where

$$d\sigma_{\text{ion}}/dW(E)$$

is the energy-loss cross section for the $i$th ionization shell (with $B_i$ its binding energy) for an incident electron of energy $E$ losing an energy equal to $W$ per unit of length, $\rho$ is the density of the traversed medium and $n$ is the number of molecules per unit of volume.

For the second term of the stopping power, which is due to electronic excitations of the target, we have

$$-(dE/dx)_{\text{exc}} = -n/\rho \sum_i \sigma_{\text{exc}}^i(E)W_i$$

where $\sigma_{\text{exc}}^i(E)$ and $W_i$ are the excitation cross section and the excitation threshold energy for the $i$th electronic excited state of the target, respectively. By substituting in Eqs. (11)–(12) the forms of cross sections.
presented in the previous paragraph (Eqs. (1)–(7)–(8)), the electron stopping power was calculated in the incident energy range from 10 eV to 1 MeV.

In Fig. 6(a), the calculated mass stopping power values for nitrogen are presented. These values are compared with the semi-empirical formula of Peterson and Green [89], the Gümüş model [90], and the values from the NIST ESTAR database [43], based on ICRU report 37 [21]. For energies higher than 30 eV, the results show agreement within 10% with the data predicted by Peterson and Green, as well as those by Majeed and Strickland. It should be noted that our results slightly overestimate the stopping power in comparison to the NIST values. This discrepancy is likely due to the RBEB model overestimating the impact ionization cross sections, in contrast to the recommended values by Itikawa (as discussed in Bug et al. [85]). Nevertheless, the agreement between our results and the NIST values remains good, with differences within 6% across the entire energy range. For energies lower than 30 eV there seems to be a lack of contribution to energy loss. To improve the agreement, a few corrections and extensions to electronic excitation states could be applied, as discussed by Grosswendt et al. [83].

In Fig. 6(b), the mass stopping power values for oxygen are compared with Peterson and Green’s semi-empirical formula, Gümüş’s model [90], Gupta’s results [91], energy loss measurements from Majeed [82], and values from NIST’s ESTAR database. A very good agreement is observed with the recommended values by NIST, with relative differences of 3.5% across the entire energy range. As for the other stopping power data, better agreement is obtained with Gupta and Majeed’s predictions, showing relative differences of about 5% for energies ranging from 30 eV to 1 keV, and less than 10% for higher energies. It could be noted that the calculated stopping power is significantly underestimated for energies below 20–30 eV, and the same consideration made in the case of nitrogen could be applied.

The stopping power values obtained by Gumus are in good agreement with the intermediate and high energy data in the ESTAR database and seems to better reproduce energy loss in the very low energy range, given also the good agreement with Peterson’s results. Nevertheless, Gumus’ model is not derived from cross-section models but is based on a modified version of Rohrllich and Carlson’s formula for collision stopping power [92,93]. This makes it inapplicable for simulation purposes. In this context, the results obtained with the proposed models in the analyzed energy range represent an excellent set of cross sections to be used for Monte Carlo applications.

4. Conclusions and perspectives

The ions produced by cosmic rays in different ionization states and spatial distribution can significantly change chemical reaction rates by orders of magnitude. Physics models for electron impact on oxygen and nitrogen molecules to be used in event-by-event Monte Carlo simulations are a necessary starting point to calculate the ionization state, the concentration and the spatial distribution of the ions produced by cosmic rays interaction with molecules in the atmosphere. In this paper, we have presented models (elastic scattering, electronic excitation and ionization processes) for electron transport in molecular oxygen and nitrogen that are applicable over a wide energy range (10 eV - 1 MeV).

We evaluated these cross section models by comparing them with experimental data, obtaining an overall good agreement. A second validation was performed by comparing the analytically calculated stopping power with values from the NIST database and from other calculation methodologies. The good agreement of the stopping power results demonstrates the applicability of the cross-section models across the entire energy range studied here.

Further work is underway to exploit these new cross sections in Monte Carlo code using a Track-Structure approach for simulation applications. Specifically, we are working on integrating these models into Geant4-DNA to simulate the ionization effects in small volumes at different altitudes in the Earth’s troposphere and stratosphere.

The extension of Geant4-DNA to simulate physics for any molecule of climatological interest opens up for the first time the possibility to accurately simulate the complicated physicochemical processes involved in the atmosphere. Besides atmospheric applications, having a complete set of low-energy electromagnetic interactions with gaseous molecules may also be of great interest for various applications such as modeling discharge phenomena, radiation chemistry, micro and nano dosimetry experiments, and exobiology studies.
Fig. 5. Cross section for electrons in nitrogen (a) and oxygen (b) plotted as a function of electron incident energy for elastic scattering (green), electron excitation (blue), ionization (red), and total (black). The results obtained in the present study for excitation, elastic, ionization and total cross section are presented in solid lines. These are compared with values from Grosswendt and Pszona [83], Porter et al. [46], Majeed and Strickland [82], Itikawa [48,49], Williart et al. [81], García and Blanco [87], Iga et al. [58], Shyn et al. [61,68], Bromberg [66], Daimon et al. [59], and Rapp and Englander-Golden [88]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NIST</td>
<td>National Institute of Standards and Technology</td>
</tr>
<tr>
<td>ICRU</td>
<td>International Commission on Radiation Units and Measurements</td>
</tr>
<tr>
<td>ESTAR</td>
<td>Stopping Powers and Ranges for Electrons</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>MC-TS</td>
<td>Monte Carlo Track Structure</td>
</tr>
<tr>
<td>Geant4</td>
<td>GEometry ANd Tracking</td>
</tr>
<tr>
<td>RBEB</td>
<td>Relativistic Binary Encounter Bethe</td>
</tr>
<tr>
<td>IAM</td>
<td>Independent Atom Model</td>
</tr>
<tr>
<td>AR</td>
<td>Additivity Rule</td>
</tr>
<tr>
<td>SCAR</td>
<td>Screening Corrected Additivity Rule</td>
</tr>
<tr>
<td>ELSEPA</td>
<td>Elastic Scattering of Electrons and Positrons by neutral Atoms</td>
</tr>
<tr>
<td>TCS</td>
<td>Total Cross Section</td>
</tr>
<tr>
<td>DCS</td>
<td>Differential Cross Section</td>
</tr>
</tbody>
</table>

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This project has been partially funded by Programma Operativo Nazionale (PON) “Ricerca e Innovazione” 2014–2020 (Decree 1061/2021) Action IV.5 “Dottorati su tematiche Green.”
**References**


—

**Fig. 6.** Energy dependence of the electronic mass stopping power in nitrogen (a) and oxygen (b) calculated using Eq. (10). Present calculations (solid black line) are compared with semi-empirical formula of Peterson and Green (purple) [89], Gümüş model (green) [90], semi-empirical formula from Gupta et al. (blue) [91], energy loss measurements from Majeed and Strickland [82], and data from NIST ESTAR database (red) [43]. The relative differences between present analytical results and the other data is shown in the bottom panel of the two figures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
F. Nicolanti et al.

Physica Medica 114 (2023) 102661

1


