9. Supplementary material

9.1 Empirical determination of solid water scaled depths

The empirical method was used to derive the $R_{80}$ ranges for each configuration tested (Table 1) from the ionisation chamber measurements (section 2.3.1.). The method can empirically determine the $R_{80}$ ranges for each configuration (Table 1) by calculating the range shift caused by using tissue-equivalent materials within the variable section of the phantom compared to the solid water only configuration. These independently derived ranges were then compared with the EBT3 film measurements (section 2.2.4.) to validate the film range results.

To understand the empirical determination of solid water scaled depth, the key equations with regards to WET$_m$ measurements are defined below. In IAEA TRS-398 report [27], the following equation for scaling the depths of plastics materials, m, to water equivalent depths, w is defined in equation 10.

$$z_w = z_m c_{pl}$$  \hspace{1cm} (10)

where $z_m$ is the depth in the plastic material (in gcm$^{-2}$), $z_w$ is the depth in water (in gcm$^{-2}$), and $c_{pl}$ is the depth scaling factor, defined by the ratio of the ranges (in g/cm$^2$) in water, $R_w^{MT}$, and in the plastic material, $R_m^{MT}$ (note that the superscript MT is used to denote the ranges are defined in mass thickness units).

$$c_{pl} = \frac{R_w^{MT}}{R_m^{MT}}$$  \hspace{1cm} (11)

Equation 10 can be converted into units of cm and can be used to approximate the WET$_m$ of a plastic material [41].

$$t_w = t_m \cdot \frac{\rho_m}{\rho_w} c_{pl} = t_m \cdot \frac{R_w}{R_m}$$  \hspace{1cm} (12)

where $t_w$ (=WET) and $t_m$ are the thicknesses (in cm) in water and in the plastic material, respectively, and $R_w$ (=$R_{80,w}^{IC}$) and $R_m$ (=$R_{80,w,m}^{IC}$) are the ranges (in cm) in water and in the plastic material, respectively.

WET$_m$ measurements have been described in several publications [32,41–43] and can be determined by

$$WET_m = R_{80,w,m}^{IC} - R_{80,w}^{IC}$$  \hspace{1cm} (13)

Equation 14 can then be substituted into equation instead of $t_w$.

$$R_{80,w,m}^{IC} - R_{80,w}^{IC} = t_m \cdot \frac{R_{80,w}^{IC}}{R_{80,w,m}^{IC}}$$  \hspace{1cm} (14)

The equation is then rearranged to determine the equation to calculate

$$R_{80,m} = \frac{R_{80,w}^{IC}}{R_{80,w,m}^{IC}}$$  \hspace{1cm} (15)

Equation 16 can be simplified to

$$R_{80,m} = \frac{R_{80,w}^{IC}}{rWET_m}$$  \hspace{1cm} (16)

Equation 16 was used to calculate the $R_{80,sw}$ range for the solid water material. This equation provides the $R_{80,sw}$ range of the proton beam through the solid water phantom configuration, determined using the ionisation chamber measurements.
To then calculate the range of the proton beam for the other phantom configurations, the solid water equivalent thickness of each individual tissue-equivalent material tested was determined by using equation 18 (by the application of equation 17).

\[
\begin{align*}
 r_{\text{WET}_m} &= \frac{t_{\text{WET}_m}}{r_{\text{WET}_m}} \\
 t_{\text{sw-eq},m} &= \frac{t_{\text{sw-eq},m}}{r_{\text{WET}_m}}
\end{align*}
\]

The solid water equivalent thickness of each material can be seen in Table 8.

**Table 8: Solid water equivalent thickness of RALPH tissue equivalent materials.** For SW 1471, the average \( r_{\text{WET}_m} \) was used. For SW 1472, the \( r_{\text{WET}_m} \) of the 0.5 cm slab was used as this sample has a density closer to the average density of the material, so provided a more accurate estimation of the average \( r_{\text{WET}_m} \) of the material.

<table>
<thead>
<tr>
<th>Material</th>
<th>( t_{\text{sw-eq},m} ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For SW 1471</td>
<td></td>
</tr>
<tr>
<td>SB5</td>
<td>8.53</td>
</tr>
<tr>
<td>AC</td>
<td>8.14</td>
</tr>
<tr>
<td>L</td>
<td>1.12</td>
</tr>
<tr>
<td>For SW 1472</td>
<td></td>
</tr>
<tr>
<td>SB5</td>
<td>7.73</td>
</tr>
<tr>
<td>AC</td>
<td>7.38</td>
</tr>
<tr>
<td>L</td>
<td>1.02</td>
</tr>
</tbody>
</table>

The solid water thickness of each heterogeneous configurations (Table 1, configuration 2-7) was then determined. For example, configuration 2 with SW 1471:

\[
t_{\text{sw-eq},\text{config}_2} = 1 \text{ cm of SW 1471} + 8.53 \text{ cm solid water equivalent thickness of SB5} + 1 \text{ cm of SW 1471}
\]

The solid water equivalent thickness of each configuration \( t_{\text{sw-eq},\text{config}_n} \) (Table 1, configurations 2-7) was then compared to the solid water only configuration, \( t_{\text{sw},\text{config}_0} \), (Table 1, configuration 1) to determine a solid water equivalent thickness difference, \( \Delta t_{\text{sw-eq},\text{config}_n} \).

\[
\Delta t_{\text{sw-eq},\text{config}_n} = t_{\text{sw-eq},\text{config}_n} - t_{\text{sw},\text{config}_0}
\]

The difference in solid water equivalent thickness was then related to the change in depth of the solid water range for the heterogeneous configuration.

\[
\Delta z_{\text{sw},\text{config}_n} = \Delta t_{\text{sw-eq},\text{config}_n}
\]

This enables the range in the heterogeneous phantom configurations to be calculated via equation 21.

\[
R_{80,\text{sw},\text{config}_n} = R_{80,\text{sw}} - \Delta z_{\text{sw},\text{config}_n}
\]
9.2. Monte Carlo simulations

9.2.1. Beam characterisation
The simulated beam was characterised against the water phantom ionisation chamber measurements and Monte Carlo results was within 2% for dose values up to 10.1 cm. The larger percentage differences in the Bragg peak region between the ionisation chamber and Monte Carlo results are due to the high dose gradient at the Bragg peak but the range is not affected (Figure 7). The beam energy was tuned to 114.79 MeV and the energy spread to FWHM=0.9 MeV.

![Figure 7: Comparison between ionisation chamber and Monte Carlo simulation data for 115 MeV proton beam in a water phantom](image)

9.2.2. Material characterisation
Table 9 shows the $I$-values used in the RALPH variable configurations simulations. These values were derived by the method described in section 2.3.3.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>$I$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW 1471</td>
<td>56.79</td>
</tr>
<tr>
<td>SW 1472</td>
<td>74.00</td>
</tr>
<tr>
<td>SB5</td>
<td>116.00</td>
</tr>
<tr>
<td>AC</td>
<td>89.00</td>
</tr>
<tr>
<td>L</td>
<td>100.60</td>
</tr>
</tbody>
</table>

*Table 9: I-values of RALPH materials derived from Monte Carlo simulations*